Loop coupled resonator optical waveguides

Junfeng Song,^{1,2,*} Lian-Wee Luo,¹ Xianshu Luo,¹ Haifeng Zhou,¹ Xiaoguang Tu,¹ Lianxi Jia,¹ Qing Fang¹ and Guo-Qiang Lo¹

¹Institute of Microelectronics, A*STAR (Agency for Science, Technology and Research), 11 Science Park Road, Science Park II, 117685 Singapore

²State Key Laboratory on Integrated opto-electronics, College of Electronic Science and Engineering, Jilin University, Changchun, 130012 China

*songjf@ime.a-star.edu.sg

Abstract: We propose a novel coupled resonator optical waveguide (CROW) structure that is made up of a waveguide loop. We theoretically investigate the forbidden band and conduction band conditions in an infinite periodic lattice. We also discuss the reflection- and transmission- spectra, group delay in finite periodic structures. Light has a larger group delay at the band edge in a periodic structure. The flat band pass filter and flat-top group delay can be realized in a non-periodic structure. Scattering matrix method is used to calculate the effects of waveguide loss on the optical characteristics of these structures. We also introduce a tunable coupling loop waveguide to compensate for the fabrication variations since the coupling coefficient of the directional coupler in the loop waveguide is a critical factor in determining the characteristics of a loop CROW. The loop CROW structure is suitable for a wide range of applications such as band pass filters, high Q microcavity, and optical buffers and so on.

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References and links

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1. Introduction

The dispersion relation of photons in periodic index distribution media exhibits band structures similar to that of electrons in crystals [1]. The periodic index distribution media introduce important phenomena and applications, such as ultra-small bend waveguides [2], filters [3], lasers [4], fibers [5], slow light [6], and so on. The reflection at the interface of different index materials in such periodic structures attributes to the constructive and destructive interference of photons. Other than optical reflection due to refractive index difference, the interference can also be realized by optical coupling effect. Microring based coupled-resonator optical waveguides (CROW) is an example of depicting such photonic band structures [7]. Utilizing thermal effect and carrier's dispersion effect to tune the resonant wavelength, CROW can be used in applications such as optical switches and modulators [8], storing light [9], optical delay line [10] and optical buffers [11].

In this paper, we propose a novel CROW structure whereby the periodic element structure is made of an optical waveguide loop. Unlike micro-ring based CROWs, the optical waveguide loops are made of a single connected waveguide which is more compact and has easier reconfiguration control than microring CROWs [12]. For better understanding of the characteristics of loop-CROW (L-CROWs), we theoretically investigated the dispersion relation of band structure in infinite lattices, the transmissivity, reflectivity and time delay characteristics in finite lattices.

This paper is organized as follows: in section 2, we discussed the optical band structure in perfect infinite L-CROWs using Broche theorem. Following this, we discussed the reflection and transmission, light phase change and group delay characteristics. Non-periodic loop lattice characteristics namely flat-top optical filter, cavity trapping light and group delay were investigated in section 3. We also analyzed the effects of optical loss using scattering matrix method is section 4. Finally, we introduced tunable coupled loop element in section 5 and concluded this paper in section 6.

2. Modeling of periodic L-CROW

Optical dispersion relation is an important characteristic for many applications. Dispersion relation of a periodic lattice can be illustrated as a photonic band structure. In this section, we focus on L-CROW with perfect lattice and analyzed their dispersion relation, reflectivity, transmissivity, and group delay time.

2.1 Infinite lattice – photonic band gap structure

Here we investigate an infinite nanowire L-CROW which is shown in Fig. 1(a). The CROW is a periodic structure where the repeat element is a loop waveguide, shown in Fig. 1(b). The light travels in four different paths, illustrated by dotted lines in Fig. 1(b). The reflectivity and transmissivity of the element are described in Eq. (1),

$$\begin{cases} r_e = 2it\kappa\exp(i\theta) \\ t_e = (t^2 - \kappa^2)\exp(i\theta) \end{cases}$$
(1)

Where t and κ are self- and cross-coupling coefficients respectively. θ is the phase change from terminal of (\bar{a}, \bar{b}) to (\bar{a}', \bar{b}') . \bar{a} and \bar{a}' denote the light transmission direction from left to right, and \bar{b} and \bar{b}' are from right to left. Generally, t and κ are determined by the coupling length and separation gap of the directional coupler. In the lossless situation, they satisfy the relation of $t^2 + \kappa^2 = 1$. They can be written as

$$\begin{aligned} t &= \cos(\varphi_{DC}) \\ \kappa &= \sin(\varphi_{DC}) \end{aligned}$$
(2)

Where φ_{DC} is the angle of directional coupler. Every L-CROW element introduces reflection and transmission. Therefore the periodic loop lattice shows similar behavior to a photonic crystal.



Fig. 1. (a) Sketch map of a L-CROW with a repeat element of a waveguide loop structure. (b) Loop element. Light is coupled from the left hand side, and four paths are shown by the dotted line. The left dotted line depicts the reflected light path, and the right dotted line depicts the transmitted light path.

According to Broche theorem, the wave function of a periodic structure can be described as a periodic function times a planar wave. Employing Eq. (2), we can get this relation as

$$\begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix} = \mathbf{T} \begin{pmatrix} \bar{a}' \\ \bar{b}' \end{pmatrix}$$
(3)

Where, **T** is a 2×2 transmission matrix and described as

$$\mathbf{T} = \begin{pmatrix} T_{1,1} & T_{1,2} \\ T_{2,1} & T_{2,2} \end{pmatrix} = \frac{1}{\cos(2\varphi_{DC})} \begin{pmatrix} \exp(-i\theta) & -i\sin(2\varphi_{DC}) \\ i\sin(2\varphi_{DC}) & \exp(i\theta) \end{pmatrix}$$
(4)

Where, θ is the light phase change for the lattice. It can be described as $\theta = lk_0 n_{eff}$. *l* is the length of a lattice, k_0 is the wave vector in vacuum and n_{eff} is the effective index of the waveguide. It is evident that $\|\mathbf{T}\| = 1$. Similar to planar photonic crystals [13], the eigenvalues Λ_{\pm} are given by Eq. (5). The two eigenvalues are reciprocal to each other. If the term in the square root of Eq. (5) is positive, i.e. Equation (6) is satisfied, the two eigenvalues are two points on an unit circle, and can be written as $\Lambda_{\pm} = exp(\pm i\vartheta)$, the ϑ is argument of Λ_{+} . This leads to the derivation of Eq. (7).

$$\Lambda_{\pm} = \frac{\cos(\theta) \pm i \sqrt{\cos^2\left(2\varphi_{DC}\right) - \cos^2\left(\theta\right)}}{\cos\left(2\varphi_{DC}\right)} \tag{5}$$

$$\cos^2\left(2\varphi_{DC}\right) \ge \cos^2\left(\theta\right) \tag{6}$$

$$\cos\vartheta = \frac{\cos(\theta)}{\cos(2\varphi_{DC})} \tag{7}$$

Conduction states exists when ϑ is a real number. Assuming that the directional coupler is independent of wavelength, we can get Eq. (8) from Eq. (6),

$$N\pi - [2\varphi_{DC}] > \theta > N\pi + [2\varphi_{DC}]$$
(8)

Where, $[2\varphi_{DC}]$ is angle $2\varphi_{DC}$ in principal value interval of $[-\pi/2, \pi/2]$, as illustrated in Fig. 2. The red dashed line is $cos^2(2\varphi_{DC})$. Areas below the dashed line satisfy the condition for conduction states (Eq. (6)). The conduction bands are denoted by grey colored regions in Fig. 2, while the forbidden regions are located between the conduction regions. The coupling strength of the directional coupler determines the width of conduction and forbidden bands. One extreme case is $cos^2(2\varphi_{DC}) = 1$, forbidden band does not exist, and conduction band exists across the entire spectrum. This corresponds to t = 0 or 1, the right two dashed loops figures in Fig. 1(b) become bent waveguides or crossed waveguides. Another extreme case is $cos^2(2\varphi_{DC}) = 0$, the conduction band does not exist, and the forbidden bands exists across the entire spectrum. This corresponds to $t^2 = \kappa^2 = 1/2$. In this case, the directional coupler is a 3-dB power splitter. From Eq. (1), we can see $t_e = 0$, every element loop are *full* reflector. From Eq. (8) we can get the conduction band width are $2[2\varphi_{DC}]$. This means that we can tune the band gap by changing the coupling coefficients of the directional couplers.



Fig. 2. Illustration of band structure. The grey regions denote the conduction bands while the white colors regions denote the forbidden bands.

2.2 Finite lattice – transmission and reflection

In the previous section, we discussed about the band structure of an infinite loop lattice. However, a more practical structure is a finite lattice. We again apply Eq. (3), the transmission relation of Nth loop are

$$\begin{pmatrix} a_N \\ b_N \end{pmatrix} = \mathbf{T}^N \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$$
(9)

With $\|\mathbf{T}\| = 1$, the \mathbf{T}^N is described by Eq. (10) [14].

$$\mathbf{T}^{N} = U_{N-1}\mathbf{T} - U_{N-2}\mathbf{I}$$
(10)

Where $U_N = sin[(N+1)\vartheta]/sin\vartheta$ is the second kind of Chebyshev polynomials. And matrix I is 2×2 unit matrix. Therefore transmissivity and reflectivity are given as

$$\begin{cases} t_N = \frac{1}{T_{1,1}U_{N-1} - U_{N-2}} \\ r_N = \frac{T_{2,1}U_{N-1}}{T_{1,1}U_{N-1} - U_{N-2}} \end{cases}$$
(11)

For the lossless situation, i.e. $|t_N|^2 + |r_N|^2 = 1$, we can get relation of

$$\begin{cases} \left| t_{N} \right|^{2} = \frac{1}{1 + \left| T_{2,1} \right|^{2} \left| U_{N-1} \right|^{2}} \\ \left| r_{N} \right|^{2} = \frac{\left| T_{2,1} \right|^{2} \left| U_{N-1} \right|^{2}}{1 + \left| T_{2,1} \right|^{2} \left| U_{N-1} \right|^{2}} \end{cases}$$
(12)

Figures 3(a) and 3(b) show the reflection and transmission spectrum with different lattice numbers. For N = 1, resonance does not exist across the entire spectrum (see the blue line in Figs. 3(a) and 3(b)). As the lattice number increases, the light oscillates between the loops, leading to the formation of the conduction- and forbidden- bands. In the conduction bands, the reflectivity and transmissivity spectrum show *oscillation* shape. And the oscillation number (zero points of $|r_n|^2$) is 2N-2 in 2π region. For $N \rightarrow \infty$, it can treat as a half infinite lattice and we can get the envelope of this spectrum that is described in Eq. (13). The transmissivity value of any infinite lattice is between 1 and $|t_{env}|^2$ while the reflectivity value of any infinite lattice is between 1 and $|t_{env}|^2$ are the envelope values of transmissivity and reflectivity respectively. These values are dependent on either $T_{2,1}$ or the coupling coefficients of the directional coupler.



Fig. 3. (a) Reflectivity spectrum with different lattice numbers. (b) Transmissivity spectrum with different lattice numbers. Black dashed lines are infinite lattice. Reflection phase and differential term vs. detune oscillation phase with different lattice numbers. (c) and (d) are reflectivity and transmissivity phases. (e) and (f) are phase and differential term, respectively. (f) is averaged by loop lattice number. t = 0.1. The evolutions of real- and imaginary- part with optical phase detune (see Media 1) gives a better understanding of the reflection and transmission's amplitude and phase variation.

$$\begin{cases} \left| t_{env} \right|^{2} = \frac{\sin^{2} \vartheta_{re}}{\sin^{2} \vartheta_{re} + \left| T_{2,1} \right|^{2}} \\ \left| r_{env} \right|^{2} = \frac{\left| T_{2,1} \right|^{2}}{\sin^{2} \vartheta_{re} + \left| T_{2,1} \right|^{2}} \end{cases}$$
(13)

Where, $\vartheta_{re} = real(\vartheta)$.

Phase is an important interferometer parameter. From Eq. (11), the reflection and transmission phases, Φ_r and Φ_t has a relationship as follows

$$\Phi_r = \Delta \Phi + \Phi_t \tag{14}$$

Where, Φ_t is a function of θ described as

$$\tan\left(\Phi_{t}\right) = \frac{\sin\left(\theta\right)U_{N-1}}{\cos\left(2\varphi_{DC}\right)T_{N}}$$
(15)

Where $T_N = cos(N\vartheta)$ is the first kind of Chebyshev polynomials. And phase difference $\Delta \Phi$ is described as

$$\Delta \Phi = angle [T_{21}U_{N-1}] = \frac{\pi}{2} \operatorname{sgn} \left[\tan\left(2\varphi_{DC}\right) \frac{\sin\left(N\vartheta\right)}{\sin\left(\vartheta\right)} \right]$$
(16)

Obviously, $\Delta \Phi$ is an oscillating function. It varies between the two values of $\pm \pi/2$. In other words, the reflection phase changes with a π step at $U_{N-1} = 0$, or $r_N = 0$.

Phases of reflectivity and transmissivity with different loop lattice numbers are shown in Figs. 3(c) and 3(d). According to Eqs. (14) and (16), the transmissivity phase is a gradual smooth moving curve. In the conduction band, the reflectivity phase shows a π step change at $r_N = 0$. But in the forbidden band, the phase curve is smooth. For better understanding of this phenomenon, we make a small movie (Media 1) to show the relationships of real- and imaginary- part of reflectivity and transmissivity with optical phase detune. For reflection, the tracks are always tangent to the y-axis at the coordinate origin point. But for transmission, the tracks are circulating around the coordinate origin point. Therefore, the transmission phase is continually increased.

In the forbidden band, the reflectivity and transmissivity phase is a linear line at N = 1 and changes to a nonlinear profile with N>1. The group delay τ is described in Eq. (17).

$$\tau = -\frac{d\Phi_r}{d\omega} = -T_c \frac{d\Phi_r}{d\theta}$$
(17)

Where ω is angular frequency of light, and $T_c = l/v_g$ is the time of light through a lattice. v_g is group velocity of light in the waveguide. Equation (17) describes that T_c is a scale of time, second differential term $d\Phi_r/d\theta$ is an amplification ratio. Since T_c is fixed, the delay time τ is determined by $d\Phi_r/d\theta$. Figures 3(e) and 3(f) show $d\Phi_r/d\theta$ with different lattice numbers. For simple comparison, transmissivity group delay is averaged by lattice number. For a small amount of loops, the group delay shows flatness. In the conduction band, the delay time curve shows ripple and some singular points. Since the reflection phase has a step increase of π at zero reflectivity, the differential term becomes δ function at that point. The δ function happens since the waveguide loss is assumed to be absent in the calculation. We will include the optical loss in section 4. At the forbidden center, phases detune $\Delta\theta \rightarrow 0$, from Eq. (15), the differential term becomes Eq. (18), and in a infinite lattice, it becomes Eq. (19). Apparently, if $sin(2\varphi_{DC})$ tends to the value of 0, $cos^2(2\varphi_{DC})$ tends to the value of 1, the light has a larger group delay. This case corresponds to a narrow forbidden band as shown in Fig. 2. The t and κ are close to 0 or 1. The loop element is a waveguide with lower reflectivity. The light must

be reflected through a large number of loop lattices. However, at forbidden band edge, the group delay can increase tremendously. See Eq. (7), the forbidden band edge, $\cos \vartheta = \pm 1$, or $\vartheta = M\pi$, leads to Eq. (20). The delay time has an almost linear relation with the number of lattice, especially at large number of N. Figure 4 shows the relationship of the group delay time vs. reflectivity/transmissivity. The number of lattice is fixed at 16 and t = 0.1. The arrows in Fig. 4 indicate the largest group delay that occurs near the edge of the forbidden band (see Figs. 3(e) and 3(f)). Large group delay is a result of light oscillation between lattices. In the conduction band, majority of the light is in the transmitted direction; minority of light is in the reflected direction. In the case of the forbidden band, this phenomenon is contrary. This means the light is non-equilibrium for two directions in both of bands. But there is close to equilibrium of the light at the edge of the bands. This results in strong light oscillation and thereby a large group delay.

$$\left. \frac{d\Phi_r}{d\theta} \right|_{\Delta\theta \to 0} = \frac{\tan\left(N\vartheta\right)}{\tan\left(\vartheta\right)} or \left| \frac{\tan\left(N\vartheta\right)}{\sin\left(2\varphi_{DC}\right)} \right| \tag{18}$$

$$\frac{d\Phi_r}{d\theta}\Big|_{\Delta\theta\to0}^{N\to\infty} = \frac{1}{\left|\sin\left(2\varphi_{DC}\right)\right|}$$
(19)

$$\left. \frac{\partial \Phi_r}{\partial \theta} \right|_{\vartheta \to M\pi} = N \frac{1 + \tan^2 \left(2\varphi_{DC} \right) \left(1 + 2N^2 \right) / 3}{1 + \tan^2 \left(2\varphi_{DC} \right) N^2}$$
(20)



Fig. 4. Relation of delay time with reflectivity. Blue and red color curves denote reflection and transmission respectively.

3. Modeling of non-periodic L-CROW

In the previous sections, we had discussed perfect periodic L-CROW. Now we will discuss non-periodic L-CROW. Non-periodic loop lattice will change the spectra of transmission and reflection significantly. In this section, we will investigate flat top band pass filter and L-CROW microcavity. Flat top band pass filter is a key component in signal processing. In fact, the forbidden band is applicable in a flat band pass filter as shown in Fig. 3(a). However, the side modes oscillations are significant. Adopting non-periodic lattice will increase the sidemode suppression ratio (SMSR). Non-periodic lattice structure can also be applied in a microcavity. It has many applications, such as narrow band pass filter, laser, trapping light, time delay and so on.

3.1 Non-periodic structure – flat top band pass filter

Practical application is preferred to possess a maximally flat transmission or maximally flat group delay. In Fig. 3, the SMSR is small with large difference in group delay around the

band edge. This problem can be improved through the synthesis of high-order bandpass filters [15]. And this idea can be applied in a loop lattice to realize same functions. However, the cross coupling coefficient of loop and microring must satisfy Eq. (21). For instance, taking a 6 order Butterworth and Bessel filter in [15], the loop's κ_{loop} is listed in Table 1. The Butterworth and Bessel filter's transmission spectra and group delay are shown in Figs. 5(a) and 5(b). Butterworth loop filter shows a maximally flat transmission spectrum while Bessel loop filter shows a nearly flat group delay. Comparing both κ_{loop} , even though there is big difference in the transmission spectra, the maximum related difference of κ_{loop} of both structures is only ~3%. This shows that the L-CROW structure is very sensitive to κ_{loop} .

$$\kappa_{loop} = \sqrt{\left(1 + \kappa_{ring}\right)/2} \tag{21}$$

Table 1. Cross coupling coefficients of loop filter

Filter type	κ_{loop}
Butterworth	(0.962, 0.824, 0.771, 0.762, 0.771, 0.824, 0.962)
Bessel	(0.966,0.827,0.749,0.750,0.791,0.851,0.976)



Fig. 5. (a) 6 order Butterworth filter, transmission and delay with phase detune. (b) 6 order Bessel filter, transmission and delay with phase detune. Blue and red color curve denote transmission and delay respectively.

3.2 Loop cavity

3.2.1 Two lattices Fabry-Pérot cavity



Fig. 6. Illustrative comparison of loop cavity and ring resonator. (a) Loop cavity and (b) Add/drop ring resonator.

The analogy of a two lattice Fabry-Pérot loop cavity and an add/drop ring resonator are illustrated in Fig. 6. In contrast to an add/drop ring resonator, in loop cavity light travels through the cavity waveguide (see Fig. 6 dashed region) two times while light travels through the cavity waveguide one time in a conventional ring resonator. This is favorable for active devices; the tunability efficiency is increased by two times if there is an equal phase change in both types of cavities. From Eqs. (4) and (11), the cavity transmission function is derived as

shown in Eq. (22). From Eq. (1), imaginary number *i* will introduce $\pi/2$ additional phase shift. Therefore, the resonance occurs at odd integers of π which is different from a microring resonance that occurs at even integers of π . Figure 7 shows the relationships of transmission spectrum with square of self-coupling coefficient t^2 and photon phase θ . Therefore we can get the relations of Q value with coupling coefficients as (23). When $cos^2(2\varphi_{DC})$ tends to 0, i.e. the directional coupler becomes a 3-dB splitter, the Q value is higher and the transmission spectrum becomes narrower.

$$\left|t_{2}\right|^{2} = \frac{1}{1 + 4\cos^{2}(\theta) \frac{\sin^{2}(2\varphi_{DC})}{\cos^{4}(2\varphi_{DC})}}$$
(22)

Where $\theta = \theta_{loop} + \theta_{cavity}$ is the sum of the loop phase (that is described in Eq. (1)) and cavity phase (additional phase of cavity). θ is half of phase change whereby the light travels one round in cavity. Transmission is maximum at $\theta = \pi/2$.





Fig. 7. Transmission spectrum of two loop F-P cavity.

3.2.2 Defect of L-CROW light trapping

In a perfect semiconductor, if a different element is implanted into a crystal, it will change the crystal characteristics significantly. Similarly in optical field, e.g. phase shift DFB laser, it introduces an additional phase shift in the grating. The region of additional phase shift becomes a *cavity*. In a periodic loop lattice, when an additional phase shift is introduced, it also becomes a cavity. We name this as *defect* if that cavity length is very small. A sketch map of a defect loop lattice structure is shown in Fig. 8. The left and right hand side of the structure are integral lattices. The center of the structure is a region with additional phase shift. This additional phase can be introduced by thermo-optical effect, carrier's dispersion effect, or a small additional length of passive waveguide. Additional phase is denoted by θ_c in Fig. 8. The loop lattices at both ends of the structure are two equivalent mirrors if light is in the forbidden band. Their reflectivity and transmissivity are denoted by r_{f} , t_{f} , r_b and t_b , respectively. They can be described by Eq. (11).



Fig. 8. Schematic of loop lattices with defect.

Light oscillates between the two mirrors. a_n and b_n denote the forward waves and backward waves respectively. The light energy density can be determined by $|a_n|^2 + |b_n|^2$. Assuming that the amplitude of the incident light is 1, the light energy density of cavity can be described by Eq. (24). For the lossless situation, the reflectivity is purely imaginary, and transmissivity is purely real (See Eq. (11). This means that the reflection introduces $\pi/2$ additional phase shift. For cavity mode, when light oscillates one round in the cavity, total phase change should be integer times of 2π . Therefore, half cavity phase $\theta_c = \pi/2 + M\pi$, where *M* is integer, leading to Eq. (25). If r_f , and r_b are close to 1, the energy density can be very high.

$$|a_{c}|^{2} + |b_{c}|^{2} = \frac{\left(1 + |r_{b}^{2}|\right)|t_{f}^{2}|}{\left|1 - r_{b}r_{f}\exp(2i\theta_{c})\right|^{2}}$$
(24)

$$|a_{c}|^{2} + |b_{c}|^{2} = \frac{\left(1 + |r_{b}^{2}|\right)|t_{f}^{2}|}{\left(1 - |r_{b}||r_{f}|\right)^{2}}$$
(25)



Fig. 9. Characteristics comparison of entire lattice and with defect lattice. (a), (c) and (e) are for entire lattice. (b), (d) and (f) are defect lattice. (a) and (b) shows the transmission spectrum. (c) and (d) shows the optical energy distribution in lattice with different phase detune. The energy density distribution evolves from one state in (c) to another state in (d) as the phase changes from 0 to $\pi/2$ (see Media 2). (e) and (f) shows the optical energy distribution at $\Delta \theta = 0$. The total loop lattice number is 20, and self coupling coefficient t = 0.1.

Figure 9 shows the comparison of a loop lattice without and with a defect. The defect is an additional $\pi/2$ phase shift waveguide. The phase shift can be realized by inserting a small length of waveguide, or by thermal optical effect. The total loop lattice number is fixed at 20, with the left and right hand side of the structure each having 10 loops lattice number. Figures 9(a), 9(c) and 9(e) show the simulation of the lattice without defect, while Figs. 9(b), 9(d) and 9(f) show the simulation with defect lattice. The defect introduces a transparent state at forbidden band center. Figures 9(c) and 9(d) show the light energy density distribution in the lattice with different phase detune. Note the difference in the values of the color bar in Figs. 9(c) and 9(d). For the case of the structure without defect, the light in the forbidden band cannot pass through the lattices; instead the light is reflected within the initial few lattices. But at the band edge, light can pass through more lattices before finally being reflected (see Fig. 9(c)). This explains why the light has a large time delay at the band edge. Figure 9(d)shows the defect lattice trapping the light with a very high energy density. The oscillation mode energy density distributions in lattice are shown in Figs. 9(e) and 9(f). For perfect lattice, the energy decays exponentially as the light passes through each loop lattice. For defect lattice, the energy decays exponentially from defect lattice to either ends of the lattice. The maximum energy position is at the defect. It is 27 times higher than incident light in this example. According to Eqs. (5) and (7), in forbidden band, ϑ is purely imaginary, therefore the amplitude decay between adjacent lattice is as $exp(-|\vartheta|)$. A small movie (Media 2) shows the evolution process of the spectra with cavity phase change.



Fig. 10. Comparison of differential term of with/without defect. Blue curve is without defect. Red curve is with defect.

Once the light is trapped in the cavity, the output light will show a large group delay. Figure 10 shows the comparison of group delay with/without defect. The parameters are the same as in Fig. 9. The cavity mode has 2.5 times larger group delay than that of a band edge mode. With the larger trapping time, or delay time, it enhances the light-matter interaction within the defect. This characteristic can be applied in many applications, such as laser, modulator, photodetector, sensor, and so on.

4. Effects of propagation loss - scattering matrix method

All the above discussions are based on the lossless conditions. However, in reality, there is a definite waveguide loss. It comes from materials absorption and waveguide surface roughness. Generally, n_{eff} is made of two parts, real- and imaginary- part, i.e. $n_{eff} = n_r + in_i$. Phase shift is determined by the real part n_r , and loss is determined by the imaginary part n_i . Therefore, phase θ can be rewritten as $\theta = k_0 l n_{eff} = \theta_r + i\theta_i$. In the lossless case, there is no issue using transmission matrix method for the numerical calculation. However, when the loss is taken into account, the transmission matrix method is unsuitable because the term $exp(-i\theta)$ in Eq. (4) may lead to floating number overflow. Scattering matrix method can overcome this limitation. The input and output wave's relationship Eq. (3) can be rewritten as

$$\begin{pmatrix} \bar{b} \\ \bar{a}' \end{pmatrix} = \begin{pmatrix} r_e & t_e \\ t_e & r_e \end{pmatrix} \begin{pmatrix} \bar{a} \\ \bar{b}' \end{pmatrix} = \mathbf{S} \begin{pmatrix} \bar{a} \\ \bar{b}' \end{pmatrix}$$
(26)

Where elements of matrix, r_e and t_e are the same as in Eq. (1). **S** denotes the scattering matrix. Figure (11) and Eq. (27) explain how two lattice scattering matrix can be combined into one matrix. **S**^(1,2), **S**^(2,3) and **S**^(1,3) are scattering matrix between $(a_1,b'_2)^T$ and $(b_1,a'_2)^T$, $(a_2,b'_3)^T$ and $(b_2,a'_3)^T$, $(a_1,b'_3)^T$ and $(b_1,a'_3)^T$ respectively. They are described by Eq. (27a). $\overline{\mathbf{S}}^{(1,2)}$ is the scattering matrix between $(a_1,b_2)^T$ in Eq. (27b). Where θ_2 is the phase shift between $\mathbf{S}^{(1,2)}$, $\mathbf{S}^{(2,3)}$. $(a_2,b_2)^T$ is translated by a distance from $(a'_2,b'_2)^T$. This is important for a non-periodic lattice structure. The relationship between $\overline{\mathbf{S}}^{(1,2)}$ and $\mathbf{S}^{(1,2)}$ is described in Eq. (27c). Equation (27d) gives the full form of the scattering matrix. Finally, the transmissivity, $t_N = S_{2,1}^{(1,N)}$, and reflectivity, $r_N = S_{1,1}^{(1,N)}$.



Fig. 11. Schematic of scattering layers and directions.

$$\begin{pmatrix} \bar{b}_{1} \\ \bar{a}_{2}' \end{pmatrix} = \mathbf{S}^{(1,2)} \begin{pmatrix} \bar{a}_{1} \\ \bar{b}_{2}' \end{pmatrix}, \begin{pmatrix} \bar{b}_{2} \\ \bar{a}_{3}' \end{pmatrix} = \mathbf{S}^{(2,3)} \begin{pmatrix} \bar{a}_{2} \\ \bar{b}_{3}' \end{pmatrix}, \begin{pmatrix} \bar{b}_{1} \\ \bar{a}_{3}' \end{pmatrix} = \mathbf{S}^{(1,3)} \begin{pmatrix} \bar{a}_{1} \\ \bar{b}_{3}' \end{pmatrix}$$
(27a)
$$\begin{pmatrix} \bar{b}_{1} \\ -\overline{\mathbf{S}}^{(1,2)} \begin{pmatrix} \bar{a}_{1} \end{pmatrix}$$
(27b)

$$\begin{pmatrix} 1 \\ \overline{a}_2 \end{pmatrix} = \mathbf{S}^{(1,2)} \begin{pmatrix} 1 \\ \overline{b}_2 \end{pmatrix}$$
(27b)

$$\overline{\mathbf{S}}^{(1,2)} = \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\theta_2) \end{pmatrix} \mathbf{S}^{(1,2)} \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\theta_2) \end{pmatrix}$$
(27c)

$$\mathbf{S}^{(1,3)} = \begin{bmatrix} \left(\overline{S}_{1,1}^{(1,2)} & \mathbf{0} \\ \mathbf{0} & S_{2,2}^{(2,3)} \end{bmatrix} + \begin{pmatrix} \mathbf{0} & \overline{S}_{1,2}^{(1,2)} \\ S_{2,1}^{(2,3)} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{1} & -\overline{S}_{2,2}^{(1,2)} \\ -S_{1,1}^{(2,3)} & \mathbf{1} \end{pmatrix}^{-1} \begin{pmatrix} \overline{S}_{2,1}^{(1,2)} & \mathbf{0} \\ \mathbf{0} & S_{1,2}^{(2,3)} \end{pmatrix} \end{bmatrix} (27d)$$

Figure 12 shows the simulations of reflectivity, phases and group delay with different waveguide loss. The loop lattice number is fixed at 16 in this simulation. Self coupling coefficient, t, is assumed to be 0.1. The imaginary part of phase changes from 0 to 0.01 in steps of 0.002. The reflection spectrum profile is determined by the optical interference. In the lossless condition, the reflectivity can be as low as 0 due to complete destructive interference. As a result, the extinction ratio of the reflection spectrum reduces with loss as depicted in Fig. 12(a).

The phase change of reflectivity is shown in Fig. 12(b). For conductive band and lossless situation (blue line), there is a phase change of π when reflectivity is at zero. As discussed in section 2.2, this is a singular point for the differential term. However when loss is taken into consideration, the curve is smooth and continuous with large negative differential value. The group delay is shown in Fig. 12(c) and a zoom in figure of the red dashed box is shown in Fig. 12(d). As the loss becomes smaller, the absolute of differential term becomes larger and narrower. A movie of (Media 3) illustrates the evolution of the real- and imaginary- part trace of reflections and transmissions. Unlike the lossless case, the curve of a substantial loss



structure does not pass through the origin exactly. Therefore this makes the phase change gradually instead of a step function and the extinction ratio also reduces with loss.

Fig. 12. Reflection spectrum with different absorption. Imaginary parts of phases are chosen as 0:0.002:0.01. Blue color curve denote lossless case. (a) Reflectivity. (b) Phase of reflectivity. (c) Delay time. (d) Zoom in of delay time. Media 3 shows the real- and imaginary- part evolution.



Fig. 13. Reflection characteristics comparison with different absorption in defect loop lattice. Imaginary parts of the phases are chosen as 0:0.002:0.01. Blue color curve denote the lossless case. (a) Reflectivity. (b) Phase of reflectivity. A zoom in of conduction band is inserted. (c) Zooming in of conductance band. The phase changing is in 2π range. (d) Zoom in of forbidden band. The phase change is in π range.

Figure 13 shows the reflectivity spectrum of the defect L-CROW in Fig. 8. The defect introduces an additional phase shift of $\pi/2$. Figure 13(a) shows the reflectivity with detune phase. It is similar to Fig. 12(a), the extinction ratio of the reflectivity reduces with waveguide loss. But the phase of the reflectivity is significantly different from the lossless case. The maximum phase shift is within π (see Fig. 12(b)) in the conduction band. However, for defect structure, the phase shift is within 2π (see Fig. 13(b)). And at the forbidden band center, the defect structure will introduce an additional π shift. Zoom in regions of the conduction band and forbidden band are shown in Figs. 13(c) and 13(d) respectively. As discussed in section 2.2, at reflectivity close to zero (conductive band), the phase of reflectivity increases by π . But in defect structure, the phase increases by $\sim 2\pi$ (see blue line in Fig. 13(c)). The curve is smooth and continuous for structure without defect, while the phase changes by a step of π for defect structure.

The transmission spectrum is shown in Fig. 14(a). The zoom in cavity mode is shown in Fig. 14(b). The cavity mode is more sensitive to optical loss. Delay time is shown in Fig. 14(c) and the zoom in region is shown in Fig. 14(d). The delay time reduces with increasing waveguide propagation loss. The movie (Media 4) shows the evolution trace of real- and imaginary- part for reflections and transitions with $\theta_i = 0.002$.



Fig. 14. Transmission spectrum with different absorption in defect loop lattice. Imaginary parts of phases are chosen as 0:0.002:0.01. Blue color curve denote the lossless case. (a) Transmission spectra. (b) Zoom in transmission spectra at oscillation frequency. (c) Delay time. (d) Zoom in of delay time. Media 4 shows the real- and imaginary- part with phase detune for $\theta_i = 0.002$.

5. Tunable loop

As we had discussed previously, the characteristics of L-CROW are dependent on the coupling coefficients and phase of loop. However, it is difficult to get the exact coupling coefficient and phase due to fabrication variations. Therefore, we need a tuning function to compensate the fabrication discrepancy. Figure 15(b) shows a tunable waveguide phase of loop. It can modify the oscillation wavelength. The tuning function can be realized by thermal optical effect or carrier's plasmonic dispersion effect. As we discussed in section 2.1, the coupling coefficients determine the band width. One method to control the coupling coefficients is by changing the effective index of one waveguide in the directional coupler

[16]. Another more effective method is to replace the coupling region by a tunable MZI structure (see Fig. 15(c)). Tuning can be realized by thermo-optical effect or carrier dispersion effect. The reflectivity and transmissivity of the MZI structure are described as

$$\begin{cases} r_{tunable} = 2ikt(t^2P^2 - P) \\ t_{tunable} = -2(kt)^2P^2 + P - 1 \end{cases}$$
(28)

Where $P = exp(i\Delta\theta_i) + 1$, and $\Delta\theta_i$ is the phase difference between the two MZI arms. $r_{tunable}$ and $t_{tunable}$ are functions of r_e and t_e as discussed above. Figure 16 shows the reflectivity and transmissivity of MZI-loop. Unless self coupling coefficient t is very close to 0 or 1, we can tune $r_{tunable}$ and $t_{tunable}$ in full range from 0 to 1.



Fig. 15. Sketch map of tunable loop reflector. (a) A basic loop element. (b) The loop region with heater. (c) Directional coupler replaced by MZI.



Fig. 16. Transmission and reflection with the phase difference and self-coupling coefficients. (a) Transmission. (b) Reflection.

6. Conclusion

We proposed a novel coupled resonator optical waveguide (CROW) structure whereby the repeat element is a waveguide loop. We theoretically investigated the forbidden band and conduction band conditions in infinite periodic lattices. We also analyzed the reflection- and transmission- spectra, group delay time in finite periodic structures. Scattering matrix method is used in the calculation of the effects of waveguide loss. We also proposed a tunable coupling loop waveguide to compensate for the fabrication variations since the coupling efficient is a critical factor in determining the characteristics of loop CROW. The loop CROW structure is suitable for a wide range of applications such as band pass filters, high Q microcavity, and optical buffers and so on.

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